RTG Lecture
Spectral theory of automorphic forms

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The spectral theory of automorphic forms is a central research topic at the intersection of various mathematical areas such as number theory, representation theory, hyperbolic geometry and spectral theory.

This lecture will be an introduction to the spectral theory of automorphic forms and to harmonic analysis on hyperbolic surfaces. The theory of automorphic forms provides a link between number theory and the spectral theory of the Laplace operator. We will present this relation and in particular the representation theory of $\mathrm{SL}_2(\mathbb{R})$ that is involved. We will also study the Selberg trace formula and the Selberg zeta function, which provide a further link to geometry. Time depending, we will consider a more general setting: locally symmetric spaces (of rank 1).

Some references: [1], [4], [5], [7] [6] [2], [3],

References

[1] Nicolas Bergeron, *The spectrum of hyperbolic surfaces*, Springer, 2016.

[2] Daniel Bump, *Automorphic forms and representations*, no. 55, Cambridge university press, 1998.

[3] Paul Garrett, *Modern analysis of automorphic forms by example*, vol. 1, Cambridge University Press, 2018.

[4] Henryk Iwaniec, *Spectral methods of automorphic forms*, vol. 53, American Mathematical Society, Revista Matematica Iberoamericana, 2021.

[5] Serge Lang, $\mathrm{SL}_2(\mathbb{R})$, vol. 105, Springer Science & Business Media, 1985.

[6] Peter Sarnak, *Spectral theory of automorphic forms*, `https://web.math.princeton.edu/~gyujino/Sarnak_course.pdf`.

[7] Peter Sarnak, *Spectra of hyperbolic surfaces*, Bulletin of the American Mathematical Society **40** (2003), no. 4, 441–478.